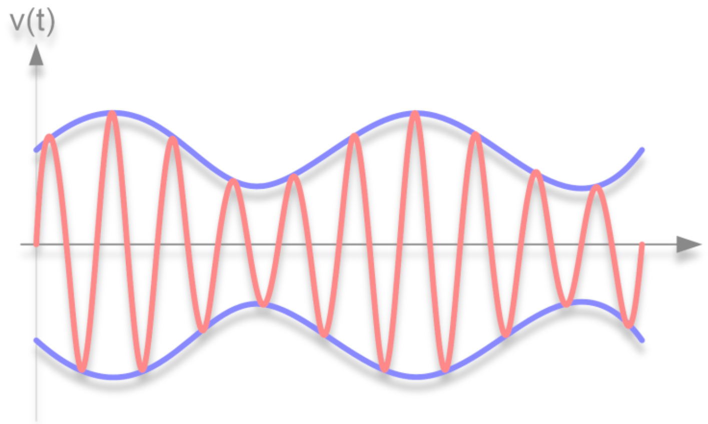
Analog Modulation

Amplitude modulation



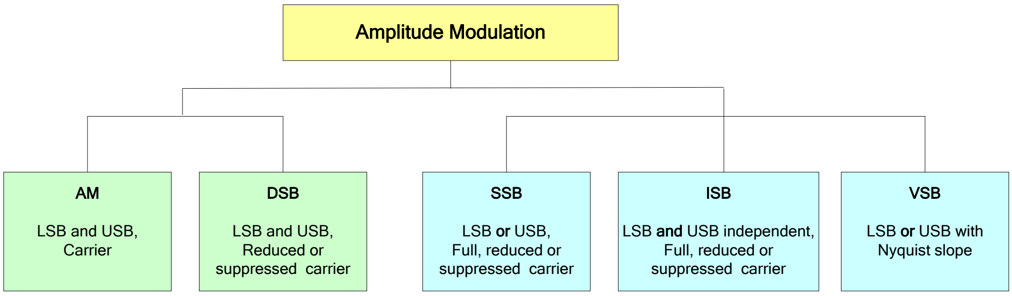
AM

Content

2. Equation Chapter 2 Section 1Analog modulation techniques

2.1 Amplitude modulation

With regard to amplitude modulation, different variants may occur:



DSB:	Double Sideband	USB:	Upper Sideband
SSB:	Single Sideband	LSB:	Lower Sideband
ISB:	Independent Sideband		
VSB:	Vestigial Sideband		

Fig. 2-1: Variants of amplitude modulations

In addition, methods with a reduced or suppressed carrier are common for DSB, SSB and ISB.

The International Telecommunication Union (ITU) designated the various types of amplitude modulation as follows:

Designation	Description
A3E	Basic AM, Double sideband, full carrier
H3E	SSB, Single sideband, full carrier
R3E	SSB, Single sideband, reduced carrier
J3E	SSB, Single sideband, suppressed carrier
B8E	ISB, Independent sideband
C3F	VSB, Vestigial sideband

Table 2-1: ITU Designations for amplitude modulation

2.1.1 Conventional amplitude modulation

The amplitude of a harmonic carrier is influenced by the modulation signal.

Carrier: $v_c(t) = \hat{V}_c \cos(\omega_c t + \varphi_c)$

Modulation Signal: $v_m(t)$

AM-Signal: $v_{AM}(t) = f(v_m(t)) \cdot \cos(\omega_c t + \varphi_c)$ (2.1)

The amplitude of an AM signal is subject to time and a function of the modulation voltage $v_m(t)$:

$$v_{c_{AM}}(t) = f(v_m(t)) \qquad (2.2)$$

This function is also designated as **Envelope**.

The frequency ω_c of the carrier does not change.

Within the context of conventional AM, the envelope simply corresponds to the sum of the unmodulated carrier amplitude $\hat{V}_c$ and the modulation signal $v_m(t)$:

$$v_{c_{AM}}(t) = \hat{V}_c + v_m(t) \tag{2.3}$$

With a sinusoidal modulation voltage

$$v_m(t) = \hat{V}_m \cdot \cos(\omega_m t) \tag{2.4}$$

the envelope will be

$$v_{c_{AM}}(t) = \hat{V}_c + v_m(t) = \hat{V}_c + \hat{V}_m \cdot \cos(\omega_m t) \tag{2.5}$$

As the carrier amplitude $\hat{V}_c$ is constant, the shape of the envelope corresponds to the modulation signal. Therefore, with $\varphi_c = 0$ and the application of the trigonometric relation

$$\cos(a)\cos(b) = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

the amplitude-modulated signal is:

$$v_{AM}(t) = v_{c_{AM}}(t)\cos(\omega_c t) = \left[\hat{V}_c + \underbrace{\hat{V}_m \cos(\omega_m t)}_{v_m(t)}\right]\cos(\omega_c t) \tag{2.6}$$

$$v_{AM}(t) = \underbrace{\hat{V}_c \cos(\omega_c t)}_{\text{Carrier}} + \frac{\hat{V}_m}{2}\left[\underbrace{\cos(\omega_c - \omega_m)t}_{\text{Lower Sideband}} + \underbrace{\cos(\omega_c + \omega_m)t}_{\text{Upper Sideband}}\right] \tag{2.7}$$

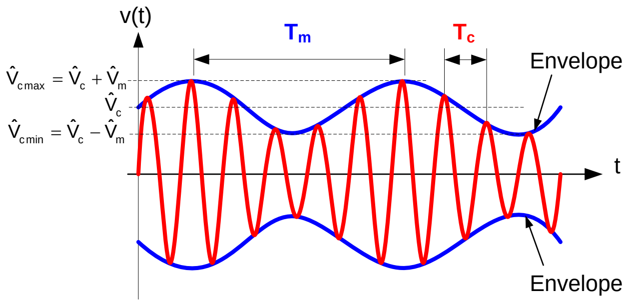


Fig. 2-2: AM- signal in time domain

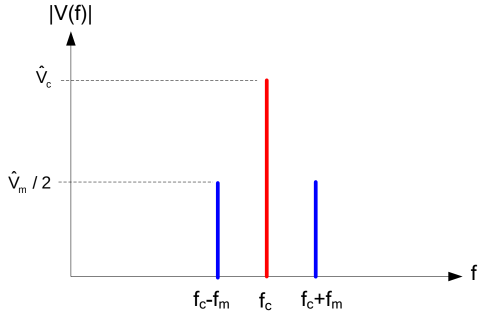


Fig. 2-3: AM- signal in frequency domain (spectrum)

The three frequency components of an AM wave with a sinusoidal modulation can also be drawn in the phase diagram as phasors rotating with angular velocity.

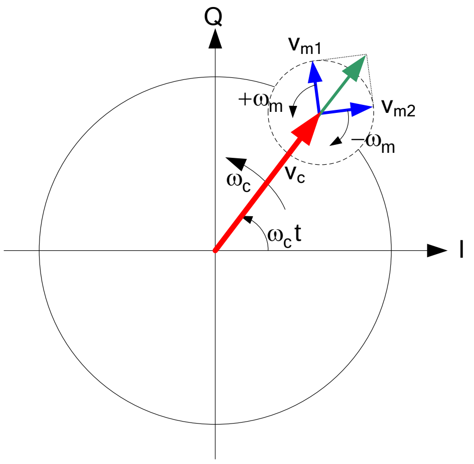


Fig. 2-4: AM signal in phase diagram (Phasor)

Referring to the carrier frequency ω_c , the phasor of the signal of the lower sideband (v_{m2}) rotates slower by ω_m , i.e. clockwise, the phasor of the signal of the upper sideband (v_{m1}) rotates faster by ω_m , i.e. anti-clockwise.

The result of the three phasors is a phasor with a varying amplitude and rotation with the carrier frequency, i.e. an amplitude-modulated signal.

Modulation index

In the equation (2.6) for the AM signal, the carrier amplitude $\hat{V}_c$ is factored out:

$$v_{AM}(t) = \left(1 + \frac{\hat{V}_m}{\hat{V}_c} \cdot \cos(\omega_m t)\right) \cdot \hat{V}_c \cdot \cos(\omega_c t)$$

Modulationindex m

$$= (1 + m \cdot \cos(\omega_m t)) \cdot \hat{V}_c \cdot \cos(\omega_c t) \qquad (2.8)$$

The modulation index m is an indicator for the degree of the amplitude modulation and is sometimes indicated in percent. After the demodulation, the amplitude of the demodulated signal is proportional to the modulation index.

$$m = \frac{\hat{V}_m}{\hat{V}_c} \quad (m \leq 1) \qquad (2.9)$$

The amplitude-modulated carrier reaches a maximum value of $\hat{V}_{c\,max} = \hat{V}_c + \hat{V}_m = \hat{V}_c\,(1+m)$ and a minimum value of $\hat{V}_{c\,min} = \hat{V}_c - \hat{V}_m = \hat{V}_c\,(1-m)$.

The modulation index m can therefore be calculated from the maximum and minimum values of the envelope:

$$m = \frac{\hat{V}_{c\,max} - \hat{V}_{c\,min}}{\hat{V}_{c\,max} + \hat{V}_{c\,min}} \tag{2.10}$$

AM signals with different modulation index

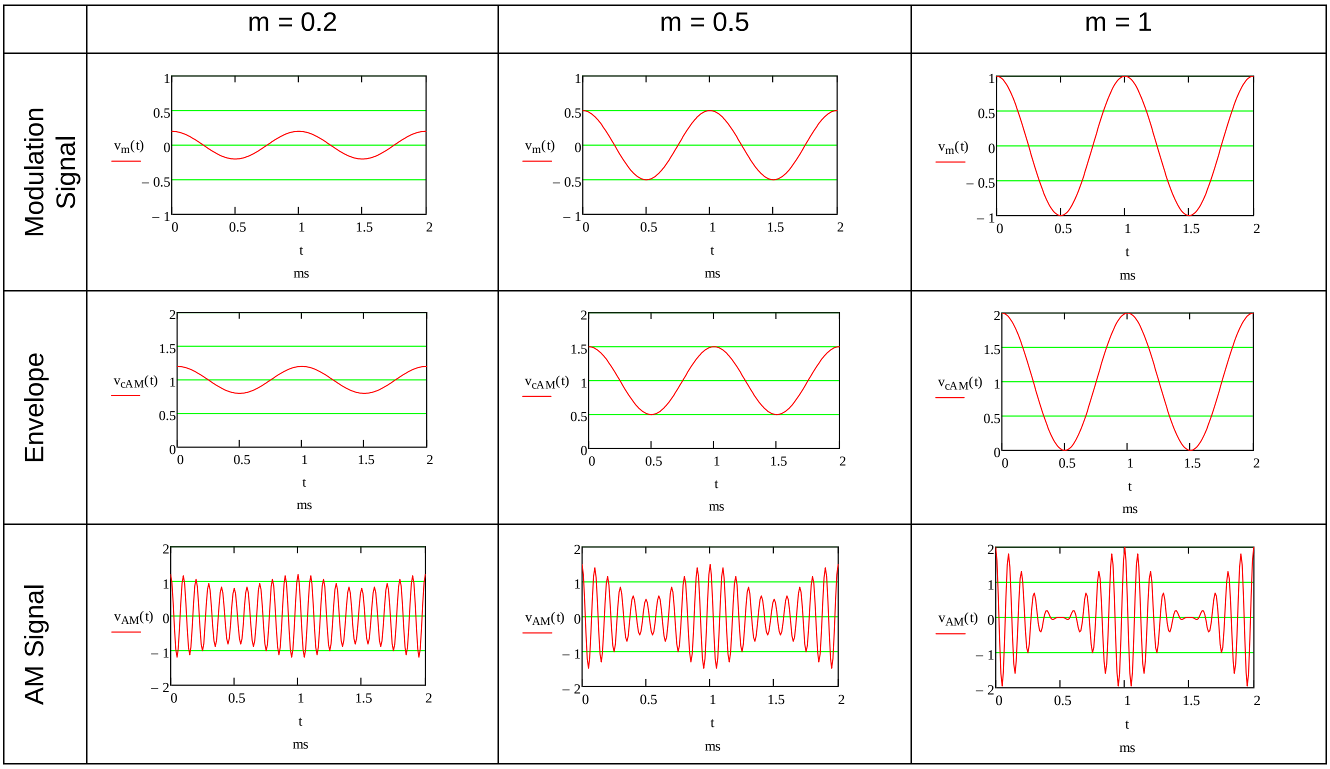


Fig. 2-5: AM- signals with modulation index of 0.2, 0.5 and 1

Overmodulation (m > 1):

In the case of $\hat{V}_m > \hat{V}_c$, the modulation index will be m>1; this is called overmodulation.

The envelopes cross the zero line. Due to this, phase jumps result, the envelope does not correspond to the modulation signal anymore and the envelope demodulation generates a distorted signal.

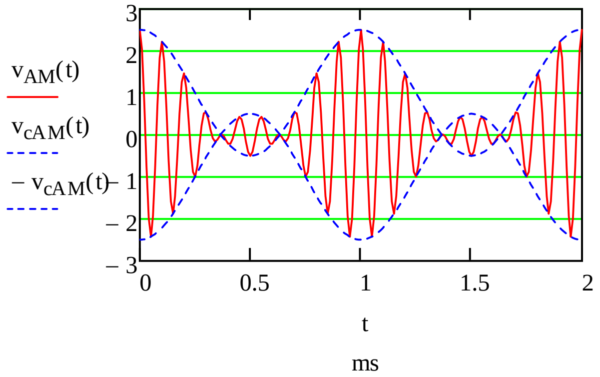


Fig. 2-6: Modulation index 1.5 (Overmodulation)

However, AM modulators are often not able to influence the phase so that the amplitude remains zero until the next phase inversion.

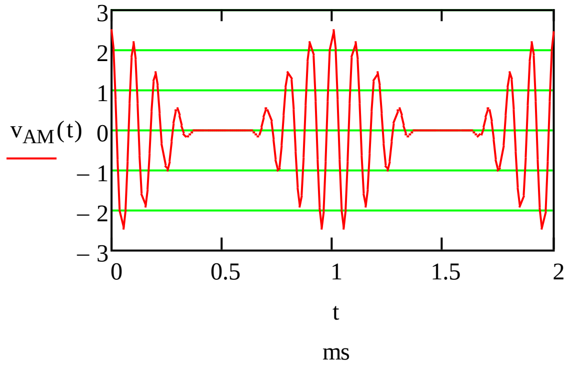


Fig. 2-7: Overmodulation without phase inversion

Amplitude spectrum of an AM signal

With the Fourier transform or by expanding and converting the equation (2.8) of the sinusoidal modulated AM signal, the individual components of the spectrum can easily be calculated:

$$\begin{aligned} v_{AM}(t) &= \left(1+m\cdot\cos(\omega_m t)\right)\cdot\hat{V}_c\cdot\cos(\omega_c t) \\ &= \underbrace{\frac{m}{2}\cdot\hat{V}_c\cdot\cos(\omega_c-\omega_m)t}_{\text{Lower Sideband}} + \underbrace{\hat{V}_c\cdot\cos(\omega_c t)}_{\text{Carrier}} + \underbrace{\frac{m}{2}\cdot\hat{V}_c\cdot\cos(\omega_c+\omega_m)t}_{\text{Upper Sideband}} \end{aligned} \tag{2.11}$$

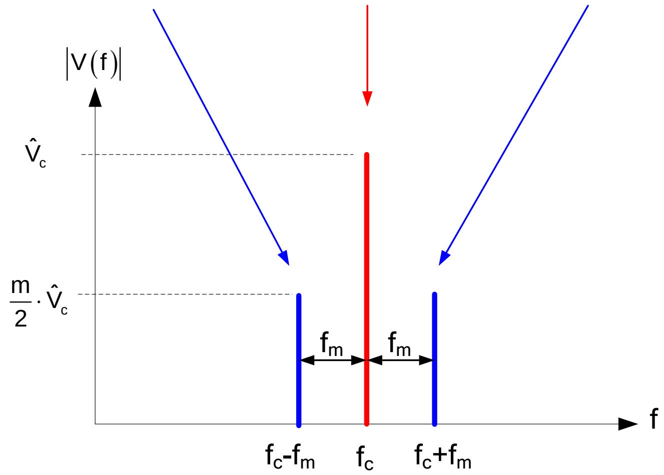


Fig. 2-8: Spectrum of AM-signal with single sinusoidal modulation signal

The amplitude spectrum consists of the following components:

- unmodulated carrier
- two sideband frequencies with a distance of the modulation frequency f_m from the carrier.

AM spectrum for random modulation signals

Non-sinusoidal modulation signals occupy a baseband in a frequency range $f_{m_{min}}$ to $f_{m_{max}}$.

In an AM signal, two sidebands are generated, which exactly correspond to the baseband, and are mirrored to the carrier frequency.

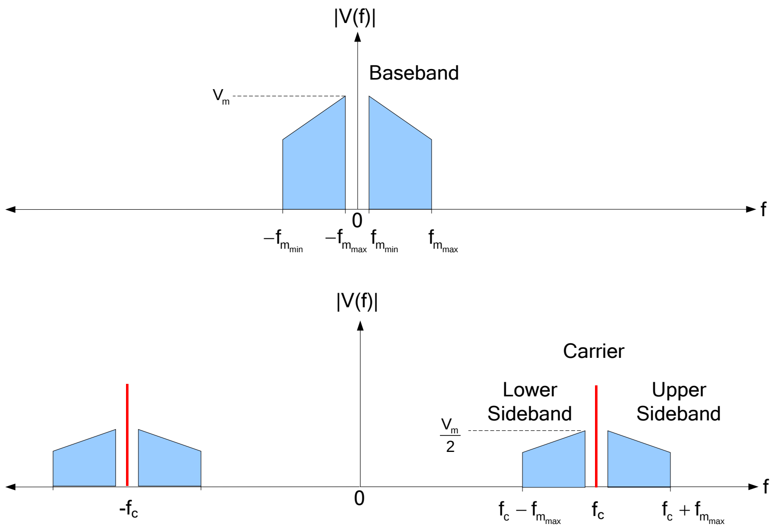


Fig. 2-9: AM-Spectrum of arbitrary modulation signals

In the lower sideband, the higher modulation frequency $f_{m_{max}}$ lies below the lower modulation frequency $f_{m_{min}}$. This is called an inverted sideband. In the upper sideband however, the relations are normal (non-inverted sideband).

Each of the sidebands respectively contains the complete information of the baseband.

Here, it is evident that due to the amplitude modulation, the double-sided baseband spectrum is shifted to the carrier frequency.

In the simplest case, a non-sinusoidal baseband signal consists of two sinusoidal components f_{m1} and f_{m2}: $u_m(t) = \hat{U}_{m1} \cdot \cos(\omega_{m1} t) + \hat{U}_{m2} \cdot \cos(\omega_{m2} t)$

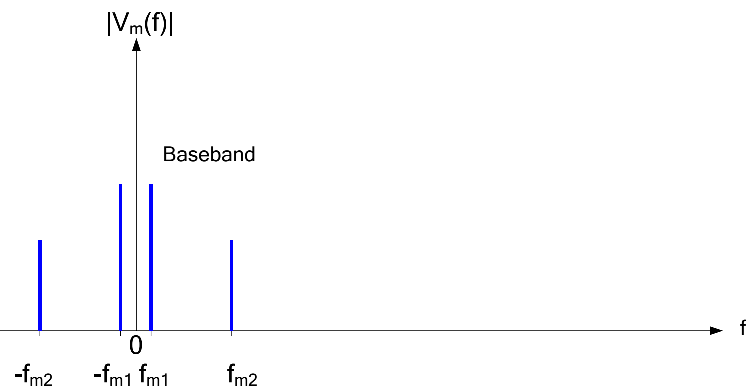


Fig. 2-10: Baseband with 2 sinusoidal modulation signals

Hence, the AM signal will be:

$$v_{AM}(t) = \left(\hat{V}_c + \hat{V}_{m1} \cdot \cos(\omega_{m1} t) + \hat{V}_{m2} \cdot \cos(\omega_{m2} t)\right) \cdot \cos(\omega_c t)$$
$$= \hat{V}_c \cdot \cos(\omega_c t) + \hat{V}_{m1} \cdot \cos(\omega_{m1} t) \cdot \cos(\omega_c t) + \hat{V}_{m2} \cdot \cos(\omega_{m2} t) \cdot \cos(\omega_c t)$$

Expanded and converted:

$$\begin{aligned} v_{AM}(t) = {} & \hat{V}_c \cdot \cos(\omega_c t) \\ & + \frac{\hat{V}_{m1}}{2} \cdot \cos\left((\omega_c - \omega_{m1})t\right) \\ & + \frac{\hat{V}_{m1}}{2} \cdot \cos\left((\omega_c + \omega_{m1})t\right) \\ & + \frac{\hat{V}_{m2}}{2} \cdot \cos\left((\omega_c - \omega_{m2})t\right) \\ & + \frac{\hat{V}_{m2}}{2} \cdot \cos\left((\omega_c + \omega_{m2})t\right) \end{aligned} \tag{2.12}$$

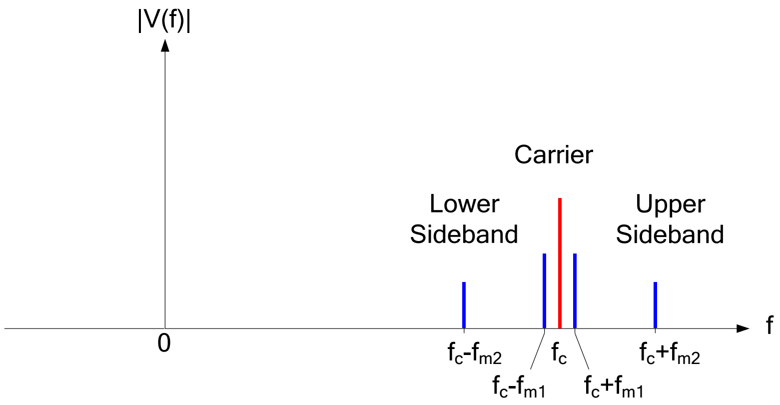


Fig. 2-11: AM Spectrum of baseband with 2 sinusoidal modulation signals

Bandwidth of AM signals

An AM signal occupies a bandwidth, which corresponds to the maximum baseband frequency multiplied by two:

$$B_{AM} = 2 \cdot f_{m_{max}} \qquad f_{m_{max}} = \text{maximum baseband frequency} \qquad (2.13)$$

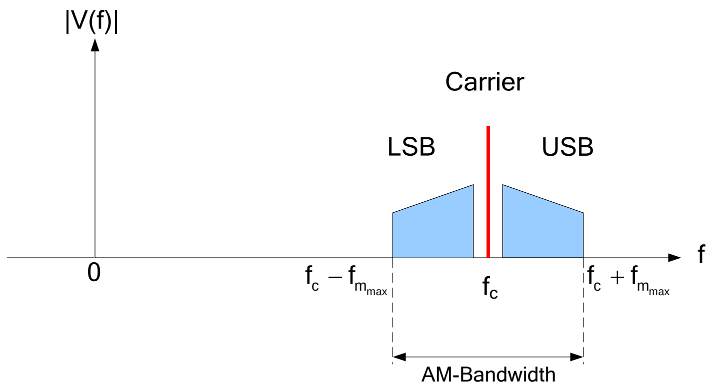


Fig. 2-12: AM Bandwidth

A receiver filter for AM has to have at least a pass bandwidth corresponding to the AM bandwidth. If the bandwidth is too small, high modulation frequencies are cut off.

Power of AM signals

The effective power realized in a resistor R is $P = \dfrac{\hat{V}^2}{2 \cdot R}$

For AM, several power definitions are used:

Carrier power = power of a unmodulated carrier $$P_c = \frac{\hat{V}_c^2}{2 \cdot R} \qquad (2.14)$$

AM power (mean power)

= sum of the powers of all frequency components. $$P_{AM} = P_c + P_{USB} + P_{OSB} \qquad (2.15)$$

$$P_{AM} = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} v_{AM}^{\;2}(t)\,dt$$

For a sinusoidal modulation signal (1-tone-modulation), this results in:

$$P_{AM} = P_c \left[1 + \left(\frac{m}{2} \right)^2 + \left(\frac{m}{2} \right)^2 \right] = P_c \cdot \left(1 + \frac{m^2}{2} \right) \qquad (2.16)$$

Peak Envelope Power PEP = power of a carrier period with maximum envelope amplitude.

For a sinusoidal modulation signal, the following applies: $$PEP = P_c \cdot (1 + m)^2 \qquad (2.17)$$

In a best-case scenario with m = 1, the mean power of an AM transmitter is 1.5 times the carrier power P_c. 2/3 of the power are located in the carrier, which does not contain any information, and only 1/3 of the power is located in the two sidebands carrying information. For the maximum values of the envelope, the transmitter must be able to generate a power that is four times the carrier power. This means high demands for the linearity of the power amplifier.

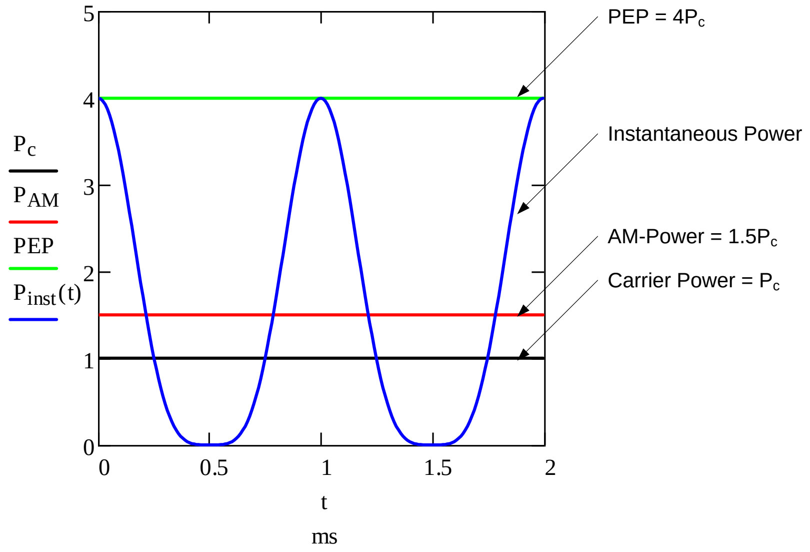


Fig. 2-13: AM-Power for 1-Tone-Modulation with f_m = 1 kHz and m = 1

2.1.2 Double Sideband Modulation DSB and Single Sideband Modulation SSB

Conventional AM signals are quite inefficient with regard to their power as well as their bandwidth. Actually, in a best-case scenario (m = 1), 2/3 of the power are located in the unmodulated carrier and only 1/3 of the power is located in the sidebands carrying the information. Therefore, it stands to reason that the carrier should at least be significantly reduced or even omitted. This leads us to double sideband modulation with a reduced carrier (DSB-RC, double sideband reduced carrier) or with a suppressed carrier (DSB-SC, double sideband suppressed carrier). In fact, DSB-SC does improve the power efficiency, however, it still has the same bandwidth as conventional AM.

As the information is completely contained in each of the sidebands, it would actually be sufficient to transmit one single sideband, which results in a single sideband modulation (SSB, single sideband). Variants:

- USB: upper sideband
- LSB: lower sideband

But in addition to the advantages in connection with these types of modulation, there is also a significantly higher circuit complexity at transmitter and receiver. Demodulation with a simple envelope demodulator is not possible.

In the past, receiving of SSB and DSB-SC signals was a very complex process and practically only possible for experienced users. In contrast to this, conventional AM offers very easy demodulation possibilities and reception is also possible for inexperienced users.

Today, these problems are not very important anymore. The fact that up to now, conventional AM has not been replaced by methods that are more efficient is however basically a question of continuity and the compatibility of systems.

Double Sideband Modulation DSB

DSB-RC

$$v_{DSB}(t) = \hat{V}_c \cdot (k + m \cdot \cos(\omega_m t)) \cdot \cos(\omega_c t)$$

DSB-SC

$$v_{DSB-SC}(t) = \hat{V}_m \cdot \cos(\omega_m t) \cdot \hat{V}_c \cdot \cos(\omega_c t)$$

Examples with sinusoidal modulation signal:

m = 1, carrier reduced to 10%

$$v_{DSB}(t) = \hat{V}_c \cdot (0.1 + m \cdot \cos(\omega_m t)) \cdot \cos(\omega_c t)$$

m = 1, carrier suppressed

$$v_{DSB-SC}(t) = \hat{V}_c \cdot (m \cdot \cos(\omega_m t)) \cdot \cos(\omega_c t)$$

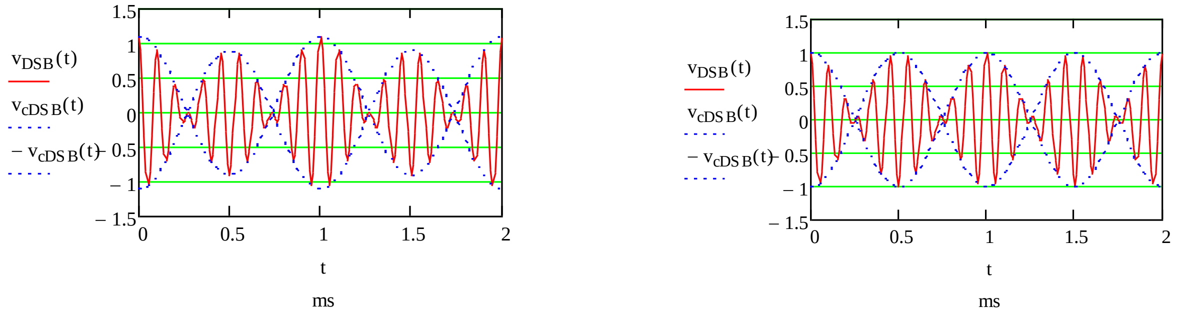


Fig. 2-14: DSB signals with reduced and suppressed carrier in time domain

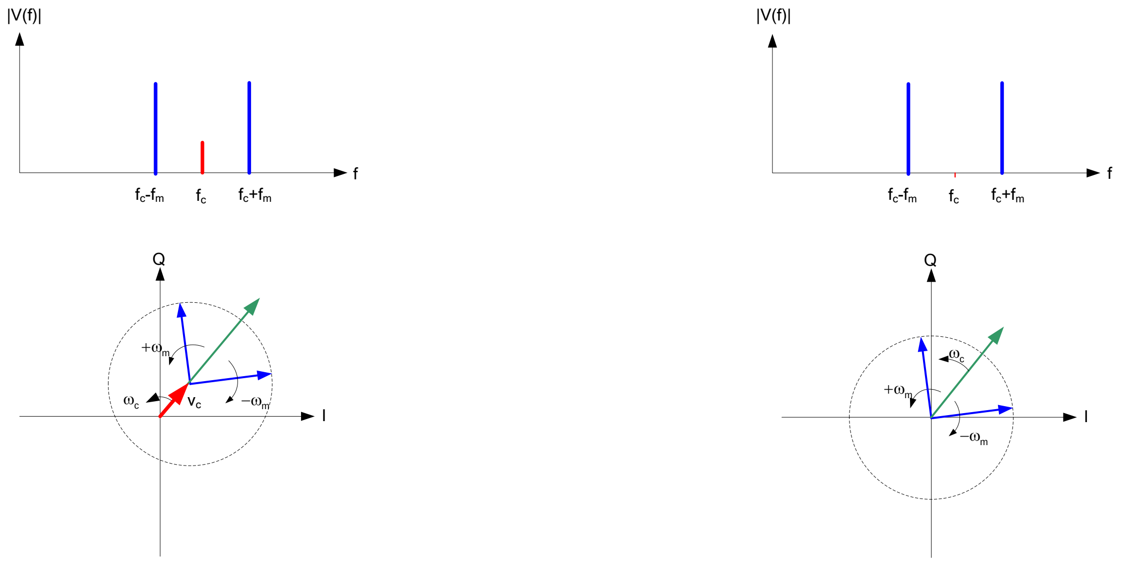


Fig. 2-15: Spectrum and phasor of DSB signals with reduced and suppressed carrier

In addition to amplitude variations, double sideband signals also show phase jumps.

In the case of the double sideband AM with a reduced or suppressed carrier, a reversal of the modulation signal $v_m(t)$ results in a phase inversion of the modulated signal and the process is not a mere amplitude modulation anymore. The form of the envelope does not correspond to the modulation signal anymore.

The power of the DSB-SC signal is calculated as follows:

$$P_{DSB-SC} = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_{AM}^{2}(t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_m^{2}(t)\hat{V}_c^{2}\cos^2(\omega_c t)\,dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_m^{2}(t)\hat{V}_c^{2}\frac{1+\cos(2\omega_c t)}{2}\,dt$$

$$= \frac{\hat{V}_c^{2}}{2}\left[\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_m^{2}(t)\,dt + \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_m^{2}(t)\cos(2\omega_c t)\,dt\right]$$

As $v_m(t)$ is a lowpass signal and its frequencies are significantly lower than $2f_c$,

$\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_m^{2}(t)\cos(2\omega_c t)\,dt = 0$ applies and therefore

$$P_{DSB-SC} = \frac{\hat{V}_c^{2}}{2}\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_m^{2}(t)\,dt = \frac{\hat{V}_c^{2}}{2}P_m$$

P_m = power of the modulation signal

Single sideband modulation SSB

Examples with sinusoidal modulation signal:

LSB, m = 1, carrier reduced to 25%

$$v_{LSB}(t) = \hat{V}_c\cdot\left(0.25\cdot\cos(\omega_c t) + \frac{m}{2}\cdot\cos(\omega_c-\omega_m)t)\right)$$

LSB, m = 1, carrier suppressed

$$v_{LSB}(t) = \hat{V}_c\cdot\frac{m}{2}\cdot\cos(\omega_c-\omega_m)t)$$

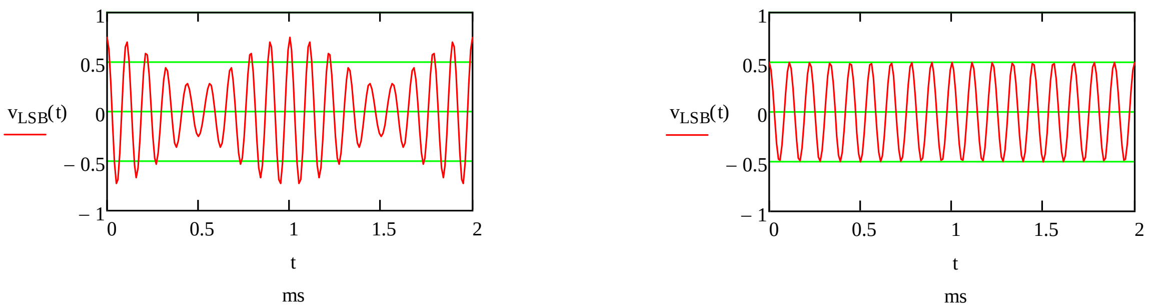


Fig. 2-16: SSB Signals for reduced and suppressed carrier, time domain

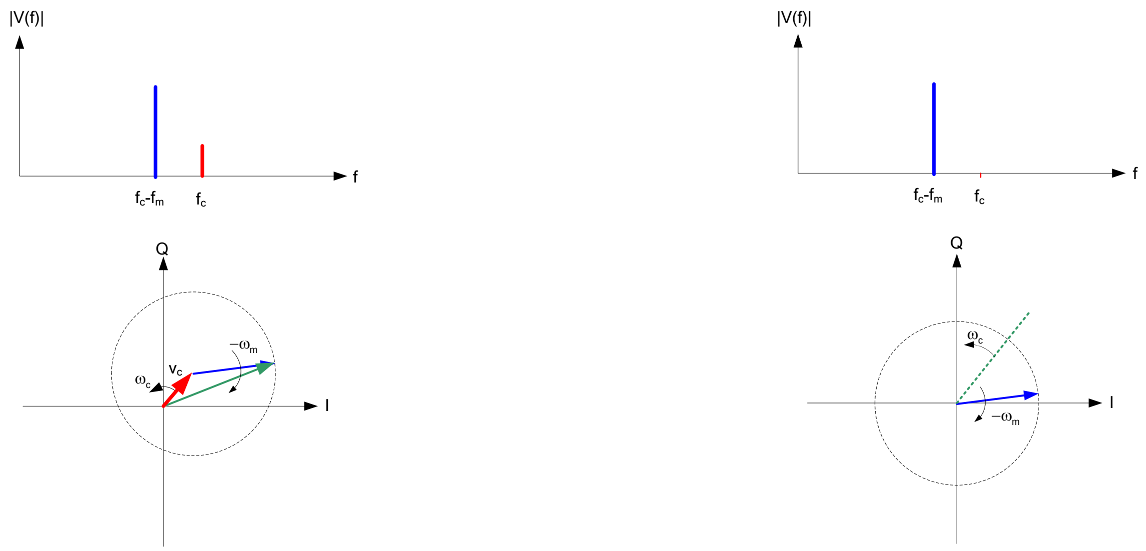


Fig. 2-17: Spectrum and phasor of SSB signals with reduced and suppressed carrier,

In the case of a single sideband signal with carrier (may be reduced), there are also phase variations.

In the case of an SSB signal with carrier or reduced carrier, the modulation signal can still be detected in the envelope. However, the envelope has a distorted shape in comparison to the modulation signal.

For the envelope of an SSB signal with carrier, the following applies:

$$v_{E\,SSB-FC}(t) = \hat{V}_c \cdot \sqrt{1 + m^2 + 2 \cdot m \cdot \cos(\omega_m t)} \qquad (2.18)$$

Example with m = 0.7 (70%) and full carrier:

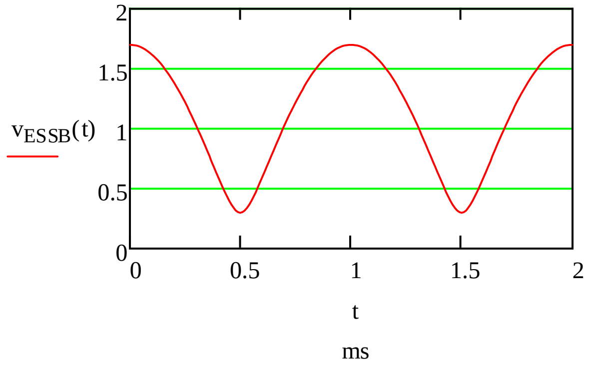


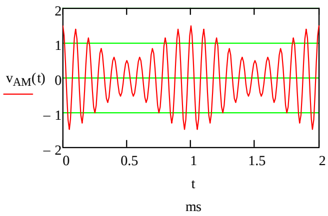
Fig. 2-18: Envelope for SSB, full carrier, m = 0.7

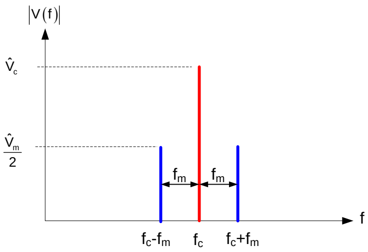
For SSB without carrier and sinusoidal modulation signal, the envelope is constant. There is no apparent modulation.

Summary

<table>
<tr><th></th><th>Double Sideband</th><th>Single (lower) Sideband</th></tr>
<tr><td>With Carrier</td><td>

$$v_{AM}(t) = \underbrace{\hat{V}_c \cos(\omega_c t)}_{\text{Carrier}} + \frac{\hat{V}_m}{2}\left[\underbrace{\cos(\omega_c - \omega_m)t}_{\text{Lower Sideband}} + \underbrace{\cos(\omega_c + \omega_m)t}_{\text{Upper Sideband}}\right]$$





Bandwidth: $B = 2 \cdot f_{m_{max}}$

Power: $P_{AM} = P_c \cdot \left(1 + \frac{m^2}{2}\right) = \frac{\hat{V}_c^2}{2 \cdot R} \cdot \left(1 + \frac{m^2}{2}\right)$

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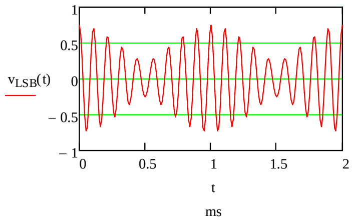
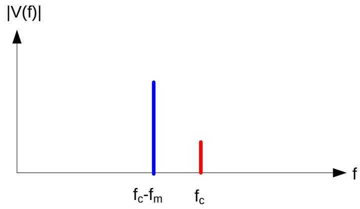
$$v_{LSB}(t) = \hat{V}_c \cdot \cos(\omega_c t) + \hat{V}_m \cos(\omega_c - \omega_m)t)$$





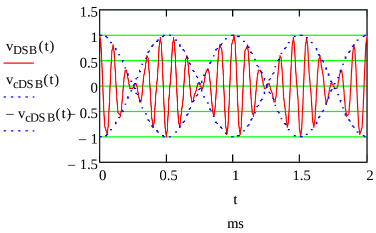
Bandwidth: $B = f_{m_{max}}$

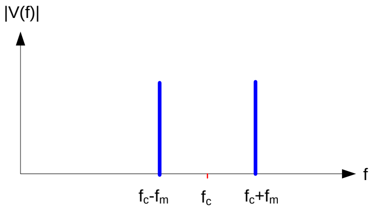
Power:

$P_{SSB-RC} = P_c \cdot (1 + m^2) = \frac{\hat{V}_c^2}{2 \cdot R} \cdot (1 + m^2)$

</td></tr>
<tr><td>Suppressed Carrier</td><td>

$$v_{DSB-SC}(t) = \frac{\hat{V}_m}{2}\left[\underbrace{\cos(\omega_c - \omega_m)t}_{\text{Lower Sideband}} + \underbrace{\cos(\omega_c + \omega_m)t}_{\text{Upper Sideband}}\right]$$





Bandwidth: $B = 2 \cdot f_{m_{max}}$

Power: $P_{DSB-SC} = \frac{\hat{V}_m^2}{4 \cdot R}$

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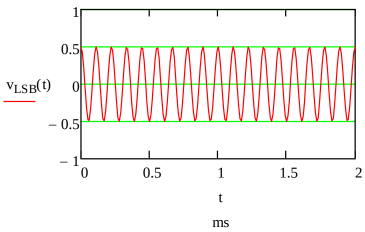
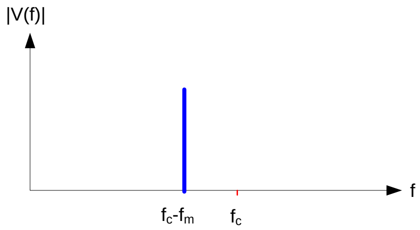
$$v_{LSB}(t) = \hat{V}_m \cdot \cos(\omega_c - \omega_m)t)$$





Bandwidth: $B = f_{m_{max}}$

Power: $P_{SSB} = \frac{\hat{V}_m^2}{2 \cdot R}$

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Fig. 2-19: Summary of amplitude modulated signals

2.1.3 Independent Sideband ISB

The two sidebands can also be modulated with different information, e.g. with the left and the right channel of a stereo signal. The practical implementation is realized by addition of an LSB and an USB signal.

2.1.4 Vestigial sideband modulation VSB

In case the baseband contains low frequencies up to 0 Hz, SSB can not or hardly be implemented (very steep filters, etc.). In this case, vestigial sideband modulation (VSB, vestigial sideband) can help, for which a small part of the unwanted sideband is included in the transmission.

In order to avoid errors, a special filter with a so-called Nyquist slope (point symmetric) has to be connected ahead of the VSB demodulation on the receiver side. The parts, which are cut off from one of the sidebands, are completed by a corresponding part of the other sideband.

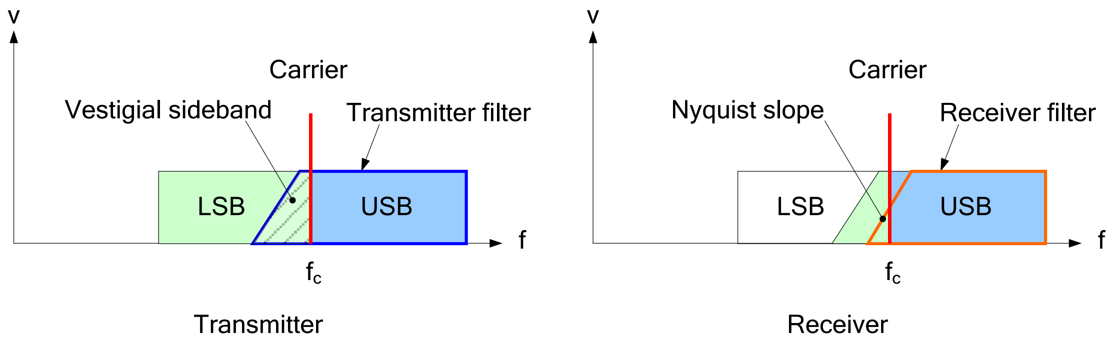

Fig. 2-20: Transmitter and receiver filter for vestigial sideband modulation

The vestigial sideband modulation is for example used in analog, terrestrial TV image transmission.

2.1.5 Quadrature AM QAM

Two carriers, which are phase-shifted against each other by 90°, can be modulated with one independent signal each and be transmitted together.

This procedure is called quadrature amplitude modulation (QAM).

The modulator according to Fig. 2-21 generates the following signal at the output

$$s_{QAM}(t) = s_{m1}(t)\hat{V}_c \cos(\omega_c t) + s_{m2}(t)\hat{V}_c \sin(\omega_c t) \tag{2.19}$$

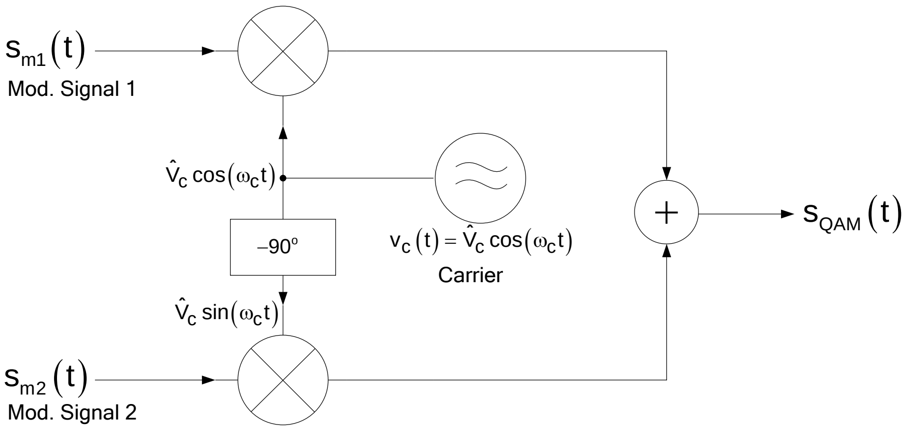

Fig. 2-21: QAM Transmitter

If this signal is transmitted to a demodulator according to Fig. 2-22 over an ideal channel, the modulation signals $s_{m1}(t)$ and $s_{m2}(t)$ are recovered with the exception of a constant amplitude factor. On the receiving side, the carrier has to be reconstructed to the exact phase with an additional circuit.

Taking into account the trigonometric identities, the signals $s_a(t)$ and $s_b(t)$ in the demodulator will be

$$\sin^2(x) = \frac{1}{2}\left[1 - \cos(2x)\right]$$

$$\cos^2(x) = \frac{1}{2}\left[1 + \cos(2x)\right]$$

$$\sin(x)\cos(x) = \frac{1}{2}\sin(2x)$$

$$\begin{aligned}
s_a(t) &= s_{QAM}(t)\hat{V}_c \cos(\omega_c t) = \left[s_{m1}(t)\hat{V}_c \cos(\omega_c t) + s_{m2}(t)\hat{V}_c \sin(\omega_c t)\right]\hat{V}_c \cos(\omega_c t) \\
&= s_{m1}(t)\hat{V}_c^2 \cos^2(\omega_c t) + s_{m2}(t)\hat{V}_c^2 \sin(\omega_c t)\cos(\omega_c t) \\
&= s_{m1}(t)\frac{\hat{V}_c^2}{2} + \underbrace{s_{m1}(t)\frac{\hat{V}_c^2}{2}\cos(2\omega_c t) + s_{m2}(t)\frac{\hat{V}_c^2}{2}\sin(2\omega_c t)}_{\text{remove with lowpass filter}}
\end{aligned}$$

$$\begin{aligned}
s_b(t) &= s_{QAM}(t)\hat{V}_c \sin(\omega_c t) = \left[s_{m1}(t)\hat{V}_c \cos(\omega_c t) + s_{m2}(t)\hat{V}_c \sin(\omega_c t)\right]\hat{V}_c \sin(\omega_c t) \\
&= s_{m1}(t)\hat{V}_c^2 \cos(\omega_c t)\sin(\omega_c t) + s_{m2}(t)\hat{V}_c^2 \sin^2(\omega_c t) \\
&= s_{m2}(t)\frac{\hat{V}_c^2}{2} + \underbrace{s_{m1}(t)\frac{\hat{V}_c^2}{2}\sin(2\omega_c t) - s_{m2}(t)\frac{\hat{V}_c^2}{2}\cos(2\omega_c t)}_{\text{remove with lowpass filter}}
\end{aligned}$$

With the lowpass filter, all signals at $2\omega_c$ are removed.

$$s_{r1}(t) = s_{m1}(t)\frac{\hat{V}_c^2}{2}$$

$$s_{r2}(t) = s_{m2}(t)\frac{\hat{V}_c^2}{2}$$

Thus, the demodulated signals $s_{r1}(t)$ and $s_{r2}(t)$ correspond to the modulations signals $s_{m1}(t)$ and $s_{m2}(t)$ of the transmitter, with the exception of the constant $\frac{\hat{V}_c^2}{2}$.

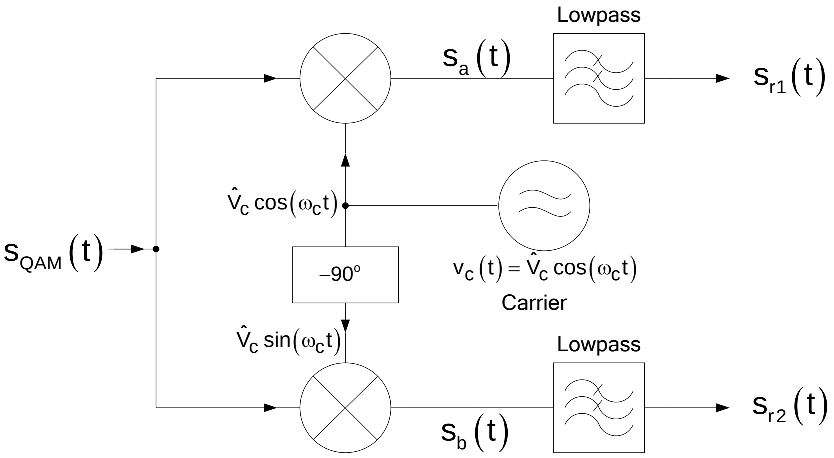

Fig. 2-22: QAM Receiver

An application of the quadrature-AM can be found at the ARI and RDS stereo broadcast.

2.1.6 AM modulators

On principle, amplitude modulation can be realized by means of adding the carrier and modulation signal and their transmission over a non-linear twoport or by means of multiplying the carrier and modulation signal.

Both methods are suitable for modulators with small signals.

Additive method

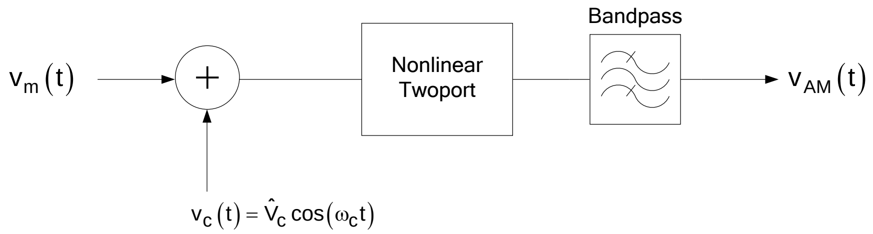

Fig. 2-23: Block diagram of additive AM modulator

Ideally, the non-linearity should have a purely square characteristic curve.

The square characteristic curve $i = k \cdot v_s^2$ is, for a sinusoidal modulation signal, controlled by the signal

$$v_s(t) = V_0 + v_m(t) + v_c(t) = V_0 + \hat{V}_m \cdot \cos(\omega_m t) + \hat{V}_c \cdot \cos(\omega_c t)$$

The current $i(t) = k \cdot \left(V_0 + \hat{V}_m \cdot \cos(\omega_m t) + \hat{V}_c \cdot \cos(\omega_c t)\right)^2$ amongst others contains the desired frequency components ω_c, $(\omega_c - \omega_m)$ und $(\omega_c + \omega_m)$.

and

For a diode modulator, the (non-linear) diode is controlled by the sum of $v_m(t)$ and $v_c(t)$ in an operating point defined by V_0. Thus, a non-linear current will flow in the circuit, which induces a voltage in the load resistor, and the spectrum of this voltage also contains the desired AM signal in addition to several unwanted signals. This signal can pass the bandpass filter.

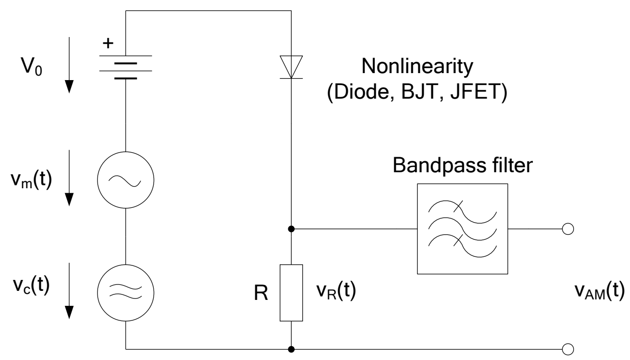

Fig. 2-24: Circuit diagram of a diode AM modulator

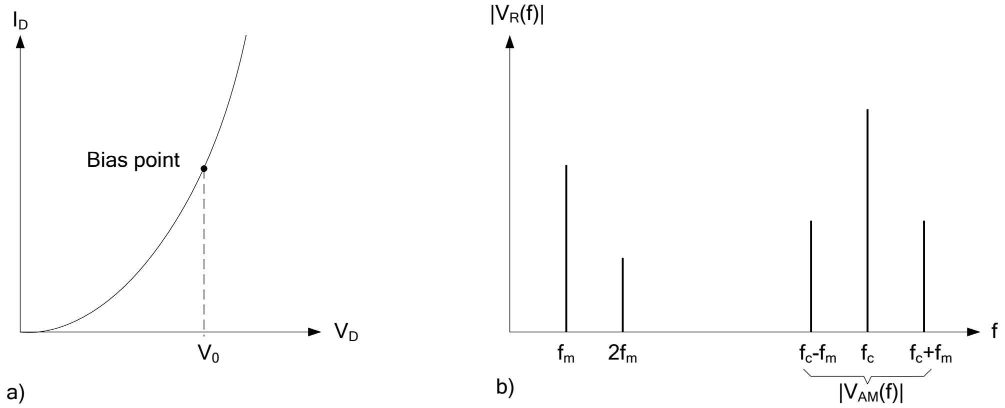

Fig. 2-25: Diode AM modulator, a) Diode IV curve, b) Spectrum of output signal

A diode only has an approximately square characteristic curve. Actually, the PN junction of a diode has an exponential curve:

$$i = I_s \cdot \left(e^{\frac{v_D}{V_T}} - 1 \right)$$

I_S = Reverse Current, V_T = Temperature Voltage (ca. 25mV), v_D = Diode Voltage

The exponential function can also be represented as a power series expansion:

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

Besides several other potentials, the square element which is necessary for the AM modulator is also contained here.

All elements of a higher order cause disturbances. They can generate unwanted frequency components, which lie within the AM band and can therefore not be removed by the bandpass filter.

The characteristic curve of a JFET has a square function, therefore, AM modulators can be realized with almost no distortion using JFET.

Multiplicative method

With the equation (2.6) $v_{AM}(t) = v_{c_{AM}}(t)\cos(\omega_c t) = \left[\hat{V}_c + v_m(t)\right]\cos(\omega_c t) = v_m(t)\cos(\omega_c t) + \hat{V}_c \cos(\omega_c t)$

two block diagrams for the realization result.

$v_{AM}(t) = \left[\hat{V}_c + v_m(t)\right]\cos(\omega_c t)$ $\qquad$ $v_{AM}(t) = v_m(t)\cos(\omega_c t) + \hat{V}_c \cos(\omega_c t)$

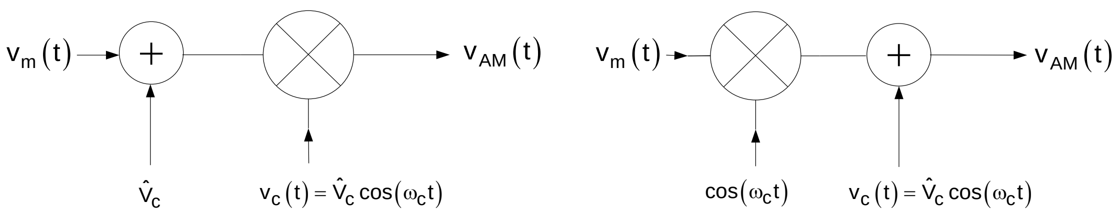

Fig. 2-26: Block diagram of multiplicative AM modulators

The block diagram on the right-hand side shows a DSB-SC modulator, where the carrier is added again. This modulator can be realized by means of the circuit in Fig. 2-27.

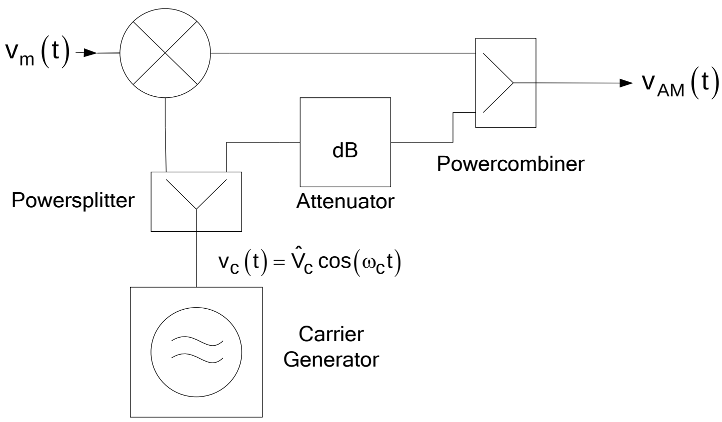

Fig. 2-27: AM modulator

AM modulators for transmitters with high power

In the case of power transmitters, the amplitude modulation is often implemented in the output power amplifier (tube or transistor). At this point, the amplification of the output amplifier is influenced by the modulation signal.

In practice, the effective supply voltage of the output amplifier is usually controlled by the modulation signal. This can for example be realized with fast switching power supply units or, as shown in the example below, by means of a modulation transformer.

The required audio frequency power lies within the range of the carrier power, as the audio signal must practically deliver the sideband power.

On the other side the power amplifier can be nonlinear class C with high efficiency. The resulting harmonics are filtered with the collector tuned circuit.

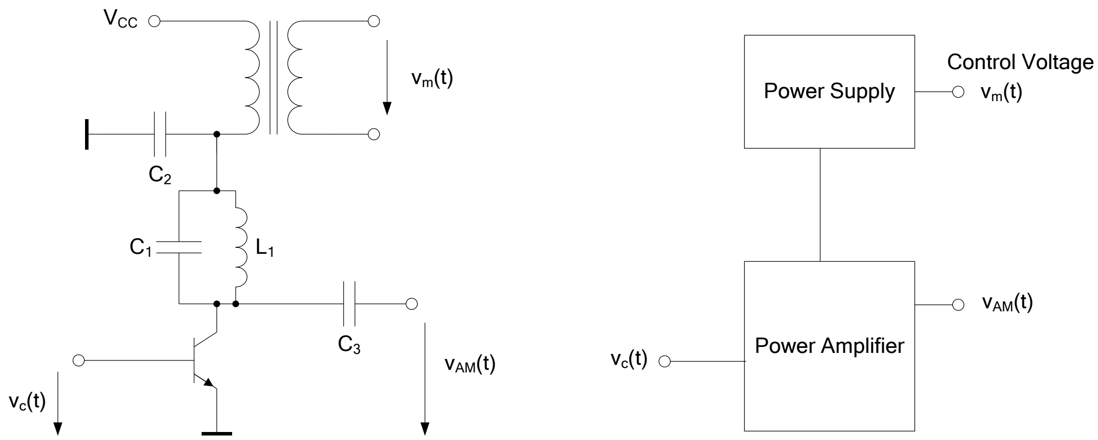

Fig. 2-28: High power AM modulators

DSB modulators

If the modulation signal is multiplied with the carrier, a double side band signal without a carrier will result (DSB-SC). As multiplicators, ring mixers according to Fig. 2-30: can be used:

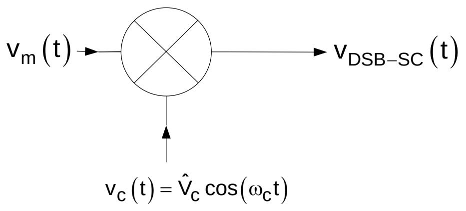

Fig. 2-29: Double sideband modulator with suppressed carrier

$$v_{DSB-SC}(t) = v_m(t) v_c(t) = \hat{V}_c \cos(\omega_c t) \cdot \hat{V}_m \cos(\omega_m t) = \frac{\hat{V}_c \hat{V}_m}{2} \left(\cos(\omega_c + \omega_m)t + \cos(\omega_c - \omega_m)t \right)$$

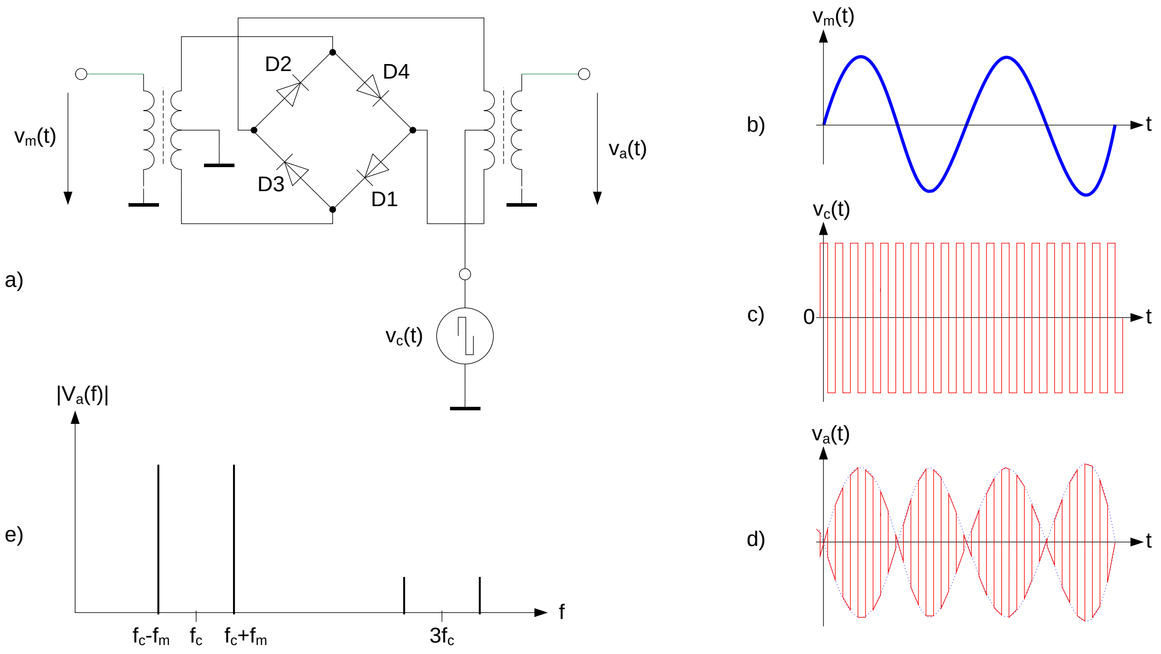

Fig. 2-30: Double balanced ring mixer, a) Circuit diagram, b) Modulation signal, c) Switching signal (Carrier), d) Output signal in time domain, e) Output spectrum

The 4 diodes switch the polarity of the modulation signals according to the carrier pulses. They can also be replaced by other switching elements, e.g. JFET or MOSFET. The most popular double-balanced mixer used in RFIC designs is the Gilbert Cell mixer.

The carrier signal $v_c(t)$ is usually a high amplitude sinusoidal signal and may be considered as a square wave signal because of the amplitude limitation by the diodes. The diodes operate as a switch.

In the positive half cycle of the carrier, the diodes D1 and D2 are conductive. Thus the output signal is $v_a(t) = v_m(t)$.

In the negative half cycle of the carrier, the diodes D3 and D4 are conductive. Thus the output signal is $v_a(t) = -v_m(t)$.

The Fourier series of the square wave is:

$$v_c(t) = \frac{4}{\pi}\left[\cos(2\pi f_c t) - \frac{1}{3}\cos(2\pi 3 f_c t) + \frac{1}{5}\cos(2\pi 5 f_c t) - ....\right]$$

and the spectrum

$$V_c(f) = \frac{2}{\pi}\left[\delta(f - f_c) + \delta(f + f_c) - \frac{1}{3}\delta(f \pm f_c) + \frac{1}{5}\delta(f \pm f_c) - .....\right]$$

The convolution of the carrier spectrum $V_c(f)$ with the modulation signal spectrum $V_m(f)$ results in the spectrum of the output signal $V_a(f)$:

$$V_a(f) = \frac{2}{\pi}V_m(f \pm f_c) - \frac{2}{3\pi}V_m(f \pm 3f_c) + \frac{2}{5\pi}V_m(f \pm 5f_c) - .....$$

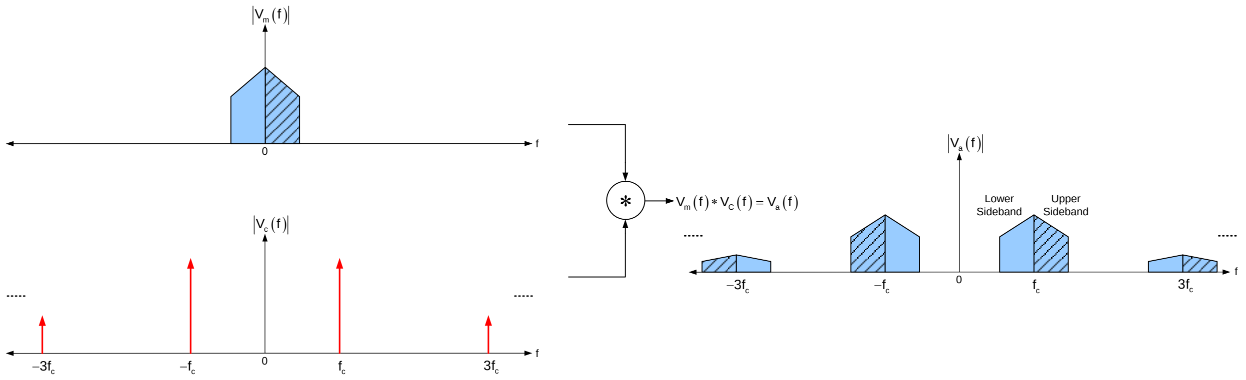

Fig. 2-31: Double balanced ringmixer, spectrum of modulation signal, carrier and output signal

With a lowpass filter the signal components at $3f_c$ and higher can be removed.

SSB modulators

Single sideband modulators according to the filter principle:

After the AM modulator (ideally a DSB-SC modulator with suppressed carrier), the unwanted sideband is removed with a narrowband bandpass filter.

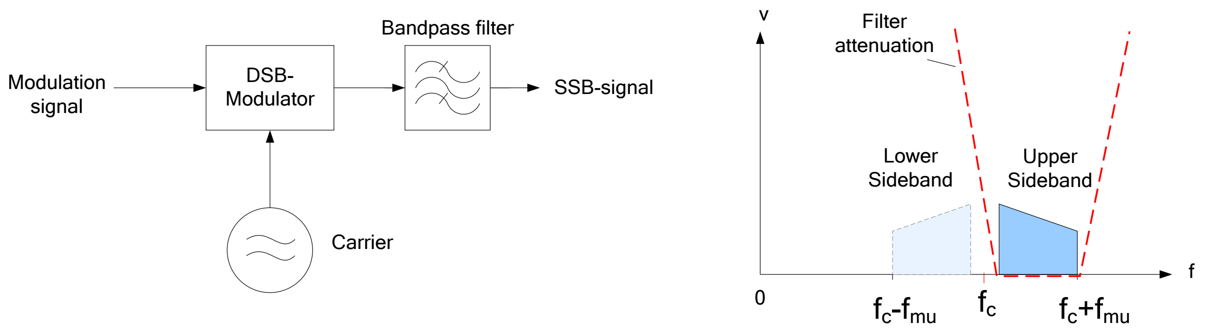

Fig. 2-32: SSB-Modulator, filter method

In order to reach the required filter slope, quartz filters are required in most applications.

The SSB modulator according to the phase method is based on the principle of the cancellation of signals in phase opposition. At the outputs of the mixers, there are both of the sidebands, however with different phase positions.

If for example, the two upper sidebands are in phase opposition, they will cancel each other out during the subsequent adding, while the two inphase lower sidebands are added. From this, an USB signal results.

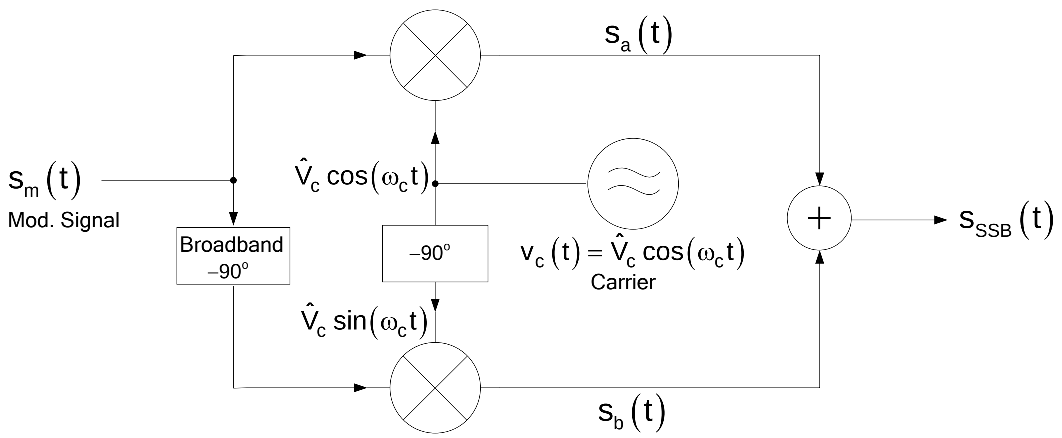

Fig. 2-33: SSB-Modulator, phase method

With analog circuit technology, the necessary broadband phase shifters can hardly be realized, so that the phase method was rarely used in the past. Today, with digital signal processing, the required circuit blocks can however be created without any problems. A broadband 90°-phaseshifter with almost ideal properties can be realized with a Hilbert transformation using digital signal processing in a DSP or FPGA.

2.1.7 AM demodulators

Incoherent demodulation

The envelope demodulator as the simplest demodulator circuit only needs a minimum number of components. It consists of a peak detector and a subsequent lowpass (RC element).

This is called an incoherent demodulation as this circuit does not require carrier recovery.

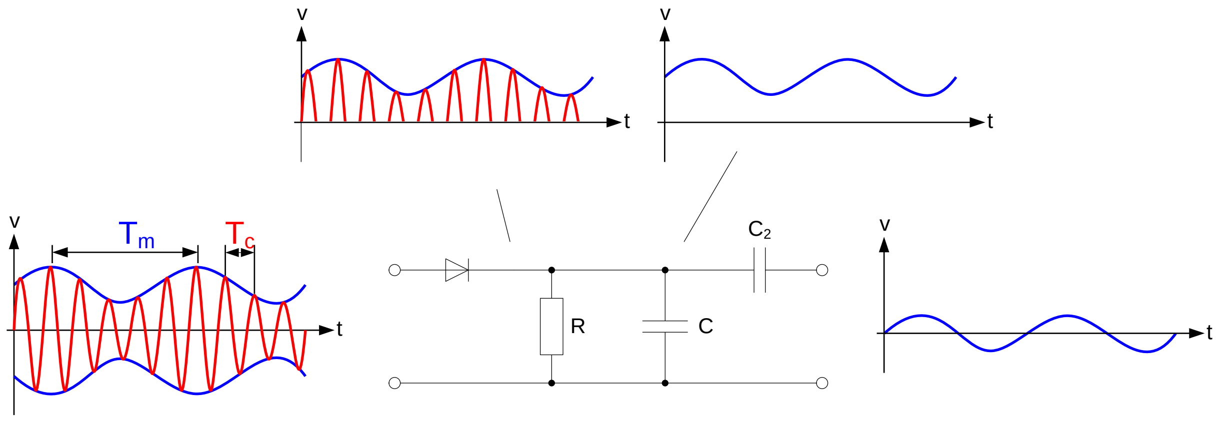

Fig. 2-34: Envelope detector, circuit and waveforms

An important factor is the selection of the time constant of the RC element. It may not be too small or too large. For its cut-off frequency f_g, the following requirement applies:

$$f_{mu} < f_g = \frac{1}{2 \cdot \pi \cdot R \cdot C} < f_c - f_{mu}$$

f_c : Carrier frequency

f_{mu} : Highest modulation frequency

As in most cases, the carrier frequency is much higher than the highest modulation frequency, the selection of the time constant is not critical.

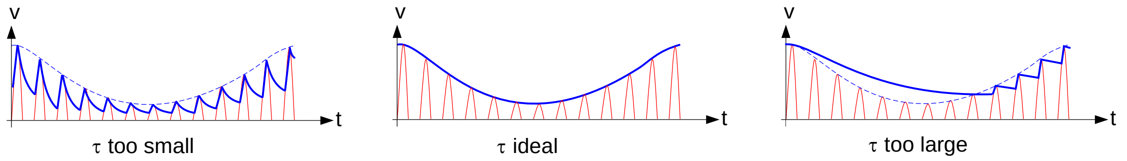

Fig. 2-35: Detector output voltage for different time constant RC

$$\tau = R \cdot C$$

The capacitor C_2 serves for DC-decoupling. Together with the load resistor of the demodulator, it forms a high-pass circuit. The cut-off frequency of the high-pass should be significantly lower than the lowest modulation frequency.

Coherent AM demodulators

The most complex but best method for AM demodulation is the product or synchronous demodulator.

The received AM signal is multiplied with a reconstructed carrier signal, i.e. the sidebands are shifted (mixed) to their original position within the baseband. After bandpass filtering, the original modulation signal is restored.

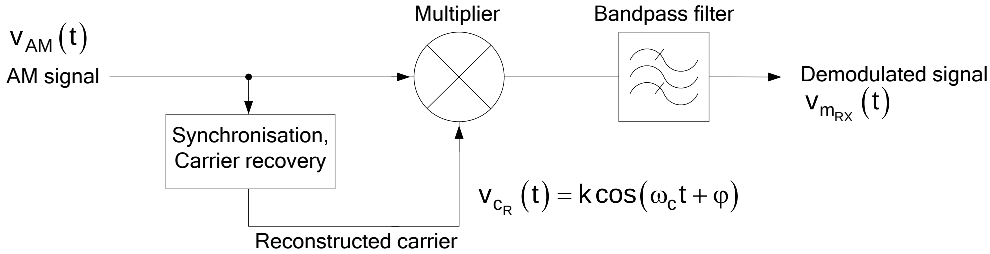

Fig. 2-36: Coherent AM demodulator, product demodulator

Analysis of the circuit:

At the input of the demodulator, an AM signal is applied $v_{AM}(t) = \left[\hat{V}_c + v_m(t)\right]\cos(\omega_c t)$

Inside the multiplier, the AM signal $v_{AM}(t)$ is multiplied with the reconstructed carrier $v_{c_D}(t) = \hat{V}_{c_D}\cos(\omega_c t + \varphi)$. Here, φ is the phase shift between the carrier of the input signal and the reconstructed carrier.

$$\begin{aligned}
v_{AM}(t)\cdot v_{c_D}(t) &= \left[\hat{V}_c + v_m(t)\right]\cos(\omega_c t)\cdot \hat{V}_{c_D}\cdot\cos(\omega_c t + \varphi)\\
&= \left[\hat{V}_c + v_m(t)\right]\frac{\hat{V}_{c_D}}{2}\left[\cos(2\omega_c t + \varphi) + \cos(\varphi)\right]\\
&= \underbrace{\frac{\hat{V}_c\hat{V}_{c_D}}{2}\left[\cos(2\omega_c t + \varphi) + \cos(\varphi)\right] + \frac{\hat{V}_{c_D}}{2}v_m(t)\cos(2\omega_c t + \varphi)}_{\text{removed by bandpass filter}} + \frac{\hat{V}_{c_D}}{2}v_m(t)\cos(\varphi)
\end{aligned}$$

$$v_{m_{RX}}(t) = \frac{\hat{V}_{c_D}}{2}v_m(t)\cos(\varphi)$$

With the exception of the factor $\frac{\hat{V}_{c_D}}{2}\cos(\varphi)$, the demodulated signal corresponds to the modulation signal of the transmitter.

In case the phase of the carrier reconstruction deviates from the phase of the transmitted carrier $\varphi \neq 0$, amplitude distortions will result. At $\varphi = \pm 90^o$, the demodulated signal will be zero. In order to facilitate a phase synchronous carrier reconstruction on the receiver side, it is better to transmit a reduced carrier.

If there is a carrier or residual carrier within the modulated signal, the original carrier can be reconstructed quite well. A phase difference with respect to the original carrier results in a DC component. Therefore, a bandpass filter is necessary on the output.

The product detector is suitable for all AM types (DSB, SSB, etc.).

For the recovery of the carrier (synchronization), a narrowband bandpass filter or a PLL (Phase Locked Loop) can be used.

If the modulation signal contains very low frequencies, the carrier recovery will be difficult with these methods.

For example, in the case of a FM stereo signal, a pilot-tone is transmitted with it and from this pilot-tone, the carrier for the demodulation can be recovered.

2.1.8 Disturbance and noise behavior of AM

In the case of AM signals, the information is carried in the variations of the amplitude. This requires linear signal processing. Non-linearities in the system chain result in non-linear distortions of the demodulated signals.

Disturbances mostly result in amplitude fluctuations. These are therefore similar to AM and can hardly be distinguished from the desired amplitude variations.

During the demodulation, the different bandwidths and power rates result in modified signal-to-noise ratios. Here, the behavior of the modulation methods with regard to noise is to be examined. The noise (AWGN Additive white Gaussian noise) is added to the wanted signal within the system. By means of a bandpass filter, a narrowband noise is generated, which has a noise power spectral density within a small bandwidth around a center frequency ω_c. This noise is often called bandpass noise. It has characteristics which correspond to a real transmission system with a narrowband receiver.

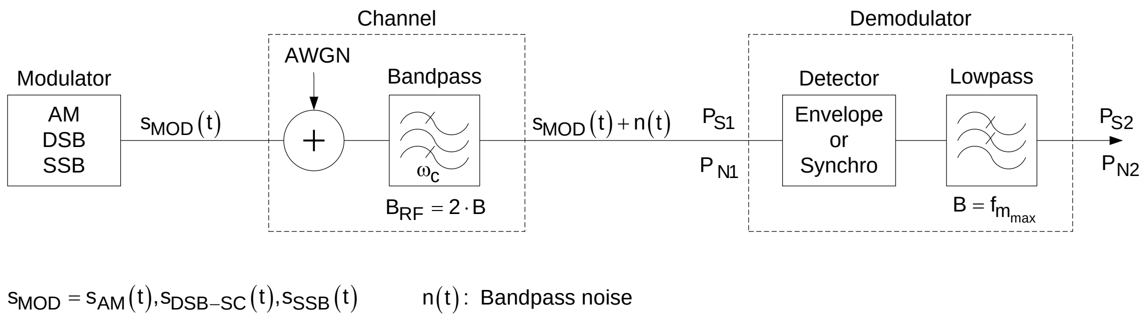

Fig. 2-37: Block diagram for noise analysis

The narrowband noise can be described with the following equation:

$$n(t) = a_n(t)\cos\left[\omega_c t + \varphi(t)\right] = n_I(t)\cos(\omega_c t) - n_Q(t)\sin(\omega_c t) \tag{2.20}$$

with $a_n(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$ and $\varphi(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$

The random variables $n_I(t)$ and $n_Q(t)$ are statistically independent from one another and not correlated. For the square mean values, the following applies:

$$\overline{n^2(t)} = \overline{n_I^2(t)} = \overline{n_Q^2(t)} \tag{2.21}$$

For the analysis, the signal-to-noise ratio at the input of the demodulator will be

$$\left(\frac{S}{N}\right)_1 = \frac{P_{S1}}{P_{N1}} \tag{2.22}$$

And at the output of the demodulator

$$\left(\frac{S}{N}\right)_2 = \frac{P_{S2}}{P_{N2}} \tag{2.23}$$

It will be analyzed for AM, DSB-SC and SSB. For AM, the envelope detector will be analyzed in addition to the synchronous detector.

With the modulation signal $s_m(t)$ the following applies to $|s_m(t)| \le 1$

$$s_{AM}(t) = S_{AM}(1+s_m(t))\cos(\omega_c t) \quad \text{AM} \tag{2.24}$$

$$s_{DSB-SC}(t) = S_{DSB-SC} s_m(t)\cos(\omega_c t) \quad \text{DSB-SC} \tag{2.25}$$

$$s_{SSB}(t) = S_{SSB}\left[s_m(t)\cos(\omega_c t) + s_m(t)\sin(\omega_c t)\right] \quad \text{SSB} \tag{2.26}$$

$S_x =$ Peak amplitude of $s_x(t)$

At the input of the demodulator in Fig. 2-37 the signal $s_{AM1}(t) = s_{AM}(t) + n(t)$ is applied to AM

$$s_{AM1}(t) = S_{AM}(1+s_m(t))\cos(\omega_c t) + n(t) \tag{2.27}$$

This results in the following signal, carrier, AM and noise power at the input of the demodulator

$$P_{S1} = \frac{1}{2} S_{AM}^2 \overline{s_m^2(t)} \tag{2.28}$$

$$P_{c1} = \frac{1}{2} S_{AM}^2 \tag{2.29}$$

$$P_{AM1} = \frac{1}{2} S_{AM}^2 \left(1 + \overline{s_m^2(t)}\right) \tag{2.30}$$

$$P_{N1} = n^2(t) = G_0 \cdot 2 \cdot B \tag{2.31}$$

Here, G_0 is the noise power spectral density, B is the bandwidth of the baseband signal and 2 B is the required RF bandwidth.

Envelope detector:

The envelope detector constitutes the envelope of the time function $s_{1AM}(t)$:

$$s_2(t) = \sqrt{\left[S_{AM}(1+s_m(t)) + n_I(t)\right]^2 + n_Q^2(t)} \tag{2.32}$$

For a large signal-to-noise ratio (low noise voltage), $s_2(t)$ can be expanded into a power series and the terms of second and higher order can be disregarded:

$$s_2(t) \approx S_{AM}(1+s_m(t)) + n_I(t) \tag{2.33}$$

At the output of the demodulator, signal and noise power will be:

$$P_{S2} = S_{AM}^2 \overline{s_m^2(t)} \tag{2.34}$$

$$P_{N2} = \overline{n_I^2(t)} = \overline{n^2(t)} = G_0 \cdot 2 \cdot B \tag{2.35}$$

With the equations (2.28), (2.31), (2.34) and (2.35), we can now calculate the signal-to-noise ratios at the input and output:

$$\left(\frac{S}{N}\right)_1 = \frac{P_{S1}}{P_{N1}} = \frac{\frac{1}{2} S_{AM}^2 \overline{s_m^2(t)}}{G_0 \cdot 2 \cdot B} = \frac{\overline{s_m^2(t)}}{G_0 \cdot 2 \cdot B} P_{c1} \tag{2.36}$$

$$\left(\frac{S}{N}\right)_2 = \frac{P_{S2}}{P_{N2}} = \frac{S_{AM}^2 \overline{s_m^2(t)}}{G_0 \cdot 2 \cdot B} = \frac{2 \cdot \overline{s_m^2(t)}}{G_0 \cdot 2 \cdot B} P_{c1} = \frac{\overline{s_m^2(t)}}{G_0 \cdot B} P_{c1} \tag{2.37}$$

Therefore, the following applies:

$$\left(\frac{S}{N}\right)_2 = 2\left(\frac{S}{N}\right)_1 \tag{2.38}$$

This means that the signal-to-noise ratio for small noise signals is 2 times or 3 dB better at the output of the demodulator than at the input of the demodulator.

If the overall AM power is used for the definition of the signal-to-noise ratio at the input of the demodulator, the following will apply:

$$\left(\frac{S}{N}\right)_{1'} = \frac{P_{AM1}}{P_{N1}} = \frac{\frac{1}{2}S_{AM}^2\left(1+\overline{s_m^2(t)}\right)}{G_0 \cdot 2 \cdot B} \tag{2.39}$$

$$\left(\frac{S}{N}\right)_2 = \frac{P_{S2}}{P_{N2}} = \frac{S_{AM}^2\overline{s_m^2(t)}}{G_0 \cdot 2 \cdot B} \tag{2.40}$$

$$\left(\frac{S}{N}\right)_2 = 2\frac{\overline{s_m^2(t)}}{1+\overline{s_m^2(t)}}\left(\frac{S}{N}\right)_{1'} \tag{2.41}$$

For the analysis of the envelope detector with a small signal-to-noise ratio (high noise voltage), the following equation results from a slightly different power series expansion of (2.32) at the output of the demodulator:

$$s_{2'}(t) \approx a_n(t) + S_{AM}\left(1+s_m(t)\right)\cos\left(\varphi(t)\right) \tag{2.42}$$

The second term of the equation can hardly be distinguished from the noise $a_n(t)$ and the demodulator does not generate a suitable output signal.

In the case of a small signal-to-noise ratio, the wanted signal is linked by multiplication to the disturbance variable $\cos\left(\varphi(t)\right)$ and with an increase of the noise voltage, the $(S/N)_2$ ratio at the demodulator output disproportionately decreases. This is defined as the threshold of the envelope demodulator.

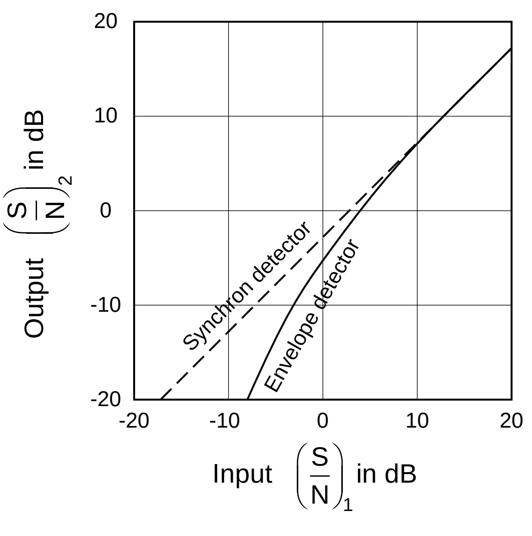

Fig. 2-38: Threshold effect of envelope detector

The transition takes place in a range of $(S/N)_1 < 10\text{dB}$ and is therefore irrelevant for most applications as signals with S/N<10dB are not of sufficient quality and cannot be used.

Further considerations can be found in Lit. [1], [2] and [5].

Synchronous detector:

In the case of the synchronous detector, the input signal of the detector $s_{1AM}(t)$ is multiplied with the carrier $\hat{V}_c \cos(\omega_c t)$. The power at the output equals

$$P_{S2} = \frac{1}{4} S_{AM}{}^2 \overline{s_m{}^2(t)} \tag{2.43}$$

$$P_{N2} = \frac{1}{4} \overline{n^2(t)} = \frac{1}{4} G_0 \cdot 2 \cdot B = \frac{1}{2} G_0 \cdot B \tag{2.44}$$

The power ratios at the input are the same as for the envelope detector.

$$\left(\frac{S}{N}\right)_1 = \frac{P_{S1}}{P_{N1}} = \frac{\frac{1}{2} S_{AM}{}^2 \overline{s_m{}^2(t)}}{G_0 \cdot 2 \cdot B} \tag{2.45}$$

$$\left(\frac{S}{N}\right)_2 = \frac{P_{S2}}{P_{N2}} = \frac{\frac{1}{4} S_{AM}{}^2 \overline{s_m{}^2(t)}}{\frac{1}{2} G_0 \cdot B} \tag{2.46}$$

Therefore, the following applies:

$$\left(\frac{S}{N}\right)_2 = 2\left(\frac{S}{N}\right)_1 \tag{2.47}$$

With the exception of the missing threshold, the signal-to-noise ratios correspond to those of the envelope detector. For DSB-SC and SSB, the same analyzes can be carried out. The results are summarized in
Table 2-2.

	AM Envelope det.	AM Synch.det.	DSB-SC	SSB
P_{S1}	$\frac{1}{2} S_{AM}{}^2 \overline{s_m{}^2(t)}$	$\frac{1}{2} S_{AM}{}^2 \overline{s_m{}^2(t)}$	$\frac{1}{2} S_{DSB-SC}^2 \overline{s_m{}^2(t)}$	$S_{SSB}^2 \overline{s_m{}^2(t)}$
P_{AM1}	$\frac{1}{2} S_{AM}{}^2 \left(1 + \overline{s_m{}^2(t)}\right)$	$\frac{1}{2} S_{AM}{}^2 \left(1 + \overline{s_m{}^2(t)}\right)$		
P_{N1}	$G_0 \cdot 2 \cdot B$	$G_0 \cdot 2 \cdot B$	$G_0 \cdot 2 \cdot B$	$G_0 \cdot B$
P_{S2}	$S_{AM}{}^2 \overline{s_m{}^2(t)}$	$\frac{1}{4} S_{AM}{}^2 \overline{s_m{}^2(t)}$		
P_{N2}	$G_0 \cdot 2 \cdot B$	$\frac{1}{4} G_0 \cdot 2 \cdot B$	$\frac{1}{4} G_0 \cdot 2 \cdot B$	$\frac{1}{4} G_0 \cdot B$
$\frac{(S/N)_2}{(S/N)_1}$	2	2	2	1
RF-Bandwidth B_{RF}	$2 \cdot B$	$2 \cdot B$	$2 \cdot B$	B

Table 2-2: Signal- and Noise-power for AM, DSB-SC and SSB

For DSB-SC, the improvement of the signal-to-noise ratio of 2 in comparison to 1 for SSB is neutralized by the double noise power at the input due to the double bandwidth.